

# SOBRE CAMBIOS DE REPRESENTACIÓN Y CONDICIONES DE FRONTERA GENERALES PARA OPERADORES DE DIRAC EN $(1+1)$ DIMENSIONES

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## Introducción

### Operadores de Dirac (lo básico)

$$\hat{h}_A = \hat{h}_A^\dagger = -i\hbar c \hat{\Gamma}_A \frac{\partial}{\partial x}, \quad (A = 1, 2, 3, 4)$$

$$\text{Dom}(\hat{h}_A) = \text{Dom}(\hat{h}_A^\dagger) \sim \left\{ \psi \in \mathcal{H}, \hat{h}_A \psi \in \mathcal{H} \oplus \text{condiciones de frontera} \right\}$$

- ▶  $x \in \Omega = [a, b]$
- ▶  $\psi = \psi(t, x) = [\psi_1(t, x) \ \psi_2(t, x)]^\top$
- ▶  $\mathcal{H} = \mathcal{L}^2(\Omega) \oplus \mathcal{L}^2(\Omega)$
- ▶  $\psi \in \mathcal{H} \Rightarrow \|\psi\|^2 \equiv \langle \psi, \psi \rangle = \int_\Omega dx \psi^\dagger(t, x) \psi(t, x) < \infty$
- ▶  $\text{Dom}(\hat{h}_A)$  es el dominio de  $\hat{h}_A$  [ funciones sobre las que  $\hat{h}_A$  puede actuar ]

A	$\hat{\Gamma}_A = \hat{\Gamma}_A^\dagger$	Operadores	Covariantes bilineales de Lorentz
1	$\hat{1}_2$	$\hat{h}_1 = -i\hbar c \hat{1}_2 \partial_x$	$C_1 = c \psi^\dagger \psi = c\rho$
2	$\hat{\alpha}$	$\hat{h}_2 = -i\hbar c \hat{\alpha} \partial_x$	$C_2 = c \psi^\dagger \hat{\alpha} \psi = j$
3	$\hat{\beta}$	$\hat{h}_3 = -i\hbar c \hat{\beta} \partial_x$	$C_3 = c \psi^\dagger \hat{\beta} \psi = cs$
4	$i\hat{\beta}\hat{\alpha}$	$\hat{h}_4 = -i\hbar c i\hat{\beta}\hat{\alpha} \partial_x$	$C_4 = c \psi^\dagger i\hat{\beta}\hat{\alpha} \psi = cw$

►  $j^\mu \equiv (j^0, j^1) = (c\rho, j) = (\text{componente temporal, componente espacial})$

►  $s$  es un escalar

►  $w$  es un pseudoescalar

►  $\hat{\Gamma}_A^2 = \hat{1}$ ;  $\hat{\Gamma}_B \hat{\Gamma}_A \hat{\Gamma}_B = -\hat{\Gamma}_A$  (para  $A \neq B$  y  $A, B = 2, 3, 4$ );  $\text{tr}(\hat{\Gamma}_A) = 0$

►  $\hat{C} = \sum_{A=1}^4 C_A \hat{\Gamma}_A$ , donde  $C_A = \frac{1}{2} \text{tr}(\hat{\Gamma}_A \hat{C})$

►  $\hat{C} = 2c \psi \psi^\dagger \Rightarrow C_A = c \psi^\dagger \hat{\Gamma}_A \psi$

- ▶  $(\hat{C}/2c\rho)^\dagger = \hat{C}/2c\rho$ ;  $(\hat{C}/2c\rho)^2 = \hat{C}/2c\rho \Rightarrow (c\rho)^2 = (cs)^2 + j^2 + (cw)^2$
- ▶  $\text{tr}(\hat{C}/2c\rho)^2 = 1$
- ▶  $\hat{C}/2c\rho$  es una matriz densidad

## Representaciones

Representación	$\hat{\alpha}$	$\hat{\beta}$	$\psi$
de Dirac (SR)	$\hat{\sigma}_x$	$\hat{\sigma}_z$	$[\varphi \ \chi]^\top$
de Weyl (WR)	$\hat{\sigma}_z$	$\hat{\sigma}_x$	$[\varphi_1 \ \varphi_2]^\top$
Supersimétrica, de Majorana (SSR)	$\hat{\sigma}_x$	$\hat{\sigma}_y$	$[\phi_1 \ \phi_2]^\top$

- ▶  $\hat{\alpha}' = \hat{S}\hat{\alpha}\hat{S}^{-1}$
- ▶  $\hat{\beta}' = \hat{S}\hat{\beta}\hat{S}^{-1}$
- ▶  $\psi' = \hat{S}\psi$

►  $\hat{S}^{-1} = \hat{S}^\dagger$

$$\text{De SR a WR} \rightarrow \hat{S} = (2)^{-1/2}(\hat{\sigma}_x + \hat{\sigma}_z)$$

$$\text{De SR a SSR} \rightarrow \hat{S} = (2)^{-1/2}(\hat{1}_2 + \hat{\sigma}_y \hat{\sigma}_z)$$

$$\text{De WR a SSR} \rightarrow \hat{S} = (2)^{-1}(i\hat{1}_2 + \hat{\sigma}_x + \hat{\sigma}_y + \hat{\sigma}_z)$$

►  $\hat{H}\psi = i\hbar \partial_t \psi; \hat{H} = -i\hbar c \hat{\alpha} \partial_x + mc^2 \hat{\beta} + U(x), \hat{H} \rightarrow \hat{h}_A = -i\hbar c \hat{\Gamma}_A \partial_x$

►  $\hat{H}'\psi' = i\hbar \partial_t \psi'; \hat{H}' = -i\hbar c \hat{\alpha}' \partial_x + mc^2 \hat{\beta}' + U(x), \hat{H}' \rightarrow \hat{h}'_A = -i\hbar c \hat{\Gamma}'_A \partial_x$

►  $\hat{\Gamma}'_A = \hat{S} \hat{\Gamma}_A \hat{S}^{-1}$

## Operadores de Dirac

### Condiciones de frontera

(a)

$$\hat{h}_1 = -i\hbar c \hat{1}_2 \partial_x (= c\hat{P} = c \hat{1}_2 \hat{p})$$

$$\langle \psi, \hat{h}_1 \xi \rangle - \langle \hat{h}_1 \psi, \xi \rangle = -i\hbar c [\psi^\dagger \xi] \Big|_a^b$$

$$[\psi^\dagger \psi] \Big|_a^b = [\varrho] \Big|_a^b = 0 \quad (\Rightarrow \varrho(b) = \varrho(a) \Rightarrow C_1(b) = C_1(a))$$

$$\psi(b) = \hat{U}_1 \psi(a) \quad [\hat{U}_1^{-1} = \hat{U}_1^\dagger]$$

$$\text{En la WR : } \begin{bmatrix} \varphi_1(b) \\ \varphi_2(b) \end{bmatrix} = \hat{U}_1 \begin{bmatrix} \varphi_1(a) \\ \varphi_2(a) \end{bmatrix}$$

►  $[f] \Big|_a^b = f(t, b) - f(t, a)$

(b)

$$\hat{h}_2 = -i\hbar c \hat{\alpha} \partial_x (= c \hat{\alpha} \hat{p})$$

$$\langle \psi, \hat{h}_2 \xi \rangle - \langle \hat{h}_2 \psi, \xi \rangle = -i\hbar c [\psi^\dagger \hat{\alpha} \xi] \Big|_a^b$$

$$c [\psi^\dagger \hat{\alpha} \psi] \Big|_a^b = [j] \Big|_a^b = 0 \quad (\Rightarrow j(b) = j(a) \Rightarrow C_2(b) = C_2(a))$$

$$\text{En la WR : } (\hat{\alpha} = \hat{\sigma}_z) \underbrace{\begin{bmatrix} \varphi_1(b) \\ \varphi_2(a) \end{bmatrix}}_{(*)} = \hat{U}_2 \begin{bmatrix} \varphi_2(b) \\ \varphi_1(a) \end{bmatrix}$$

- Este resultado (\*) surge de la teoría de extensiones auto – adjuntas de operadores simétricos de von Neumann

(c)

$$\hat{h}_3 = -i\hbar c \hat{\beta} \partial_x (= c \hat{\beta} \hat{p})$$

$$\langle \psi, \hat{h}_3 \xi \rangle - \langle \hat{h}_3 \psi, \xi \rangle = -i\hbar c \left[ \psi^\dagger \hat{\beta} \xi \right] \Big|_a^b$$

$$\left[ \psi^\dagger \hat{\beta} \psi \right] \Big|_a^b = [s] \Big|_a^b = 0 \quad (\Rightarrow s(b) = s(a) \Rightarrow C_3(b) = C_3(a))$$

Note que, en la SR,  $\hat{\beta} = \hat{\sigma}_z$ ,  $\Rightarrow$  en  $(\star)$  hacemos  $\varphi_1 \rightarrow \varphi$ ,  $\varphi_2 \rightarrow \chi$  y  $\hat{U}_2 \rightarrow \hat{U}_3$  :

$$\begin{bmatrix} \varphi(b) \\ \chi(a) \end{bmatrix} = \hat{U}_3 \begin{bmatrix} \chi(b) \\ \varphi(a) \end{bmatrix}$$

Y pasando a la WR :  $(\hat{\beta} = \hat{\sigma}_x)$   $\underbrace{\begin{bmatrix} \varphi_1(b) + \varphi_2(b) \\ \varphi_1(a) - \varphi_2(a) \end{bmatrix}} = \hat{U}_3 \underbrace{\begin{bmatrix} \varphi_1(b) - \varphi_2(b) \\ \varphi_1(a) + \varphi_2(a) \end{bmatrix}}$

(d)

$$\hat{h}_4 = -i\hbar c i\hat{\beta}\hat{\alpha} \partial_x (= c i\hat{\beta}\hat{\alpha} \hat{p})$$

$$\langle \psi, \hat{h}_4 \xi \rangle - \langle \hat{h}_4 \psi, \xi \rangle = -i\hbar c \left[ \psi^\dagger i\hat{\beta}\hat{\alpha} \xi \right] \Big|_a^b$$

$$\left[ \psi^\dagger i\hat{\beta}\hat{\alpha} \psi \right] \Big|_a^b = [w] \Big|_a^b = 0 \quad (\Rightarrow w(b) = w(a) \Rightarrow C_4(b) = C_4(a))$$

Note que, en la SSR,  $i\hat{\beta}\hat{\alpha} = \hat{\sigma}_z$ ,  $\Rightarrow$  en (\*) hacemos  $\varphi_1 \rightarrow \phi_1$ ,  $\varphi_2 \rightarrow \phi_2$  y  $\hat{U}_2 \rightarrow \hat{U}_4$ :

$$\begin{bmatrix} \phi_1(b) \\ \phi_2(a) \end{bmatrix} = \hat{U}_4 \begin{bmatrix} \phi_2(b) \\ \phi_1(a) \end{bmatrix}$$

Y pasando a la WR : ( $i\hat{\beta}\hat{\alpha} = \hat{\sigma}_y$ )

$$\underbrace{\begin{bmatrix} \varphi_1(b) - i\varphi_2(b) \\ \varphi_1(a) + i\varphi_2(a) \end{bmatrix}} = \hat{U}_4 \underbrace{\begin{bmatrix} \varphi_1(b) + i\varphi_2(b) \\ \varphi_1(a) - i\varphi_2(a) \end{bmatrix}}$$

Más condiciones de frontera (en la SR:  $\hat{\alpha} = \hat{\sigma}_x$ ,  $\hat{\beta} = \hat{\sigma}_z$ ,  $i\hat{\beta}\hat{\alpha} = -\hat{\sigma}_y$ )

$$\text{Dom}(\hat{h}_1 = -i\hbar c \hat{1}_2 \partial_x) \sim \begin{bmatrix} \varphi(b) \\ \chi(b) \end{bmatrix} = \hat{T}_1 \begin{bmatrix} \varphi(a) \\ \chi(a) \end{bmatrix}$$

$$\text{Dom}(\hat{h}_2 = -i\hbar c \hat{\alpha} \partial_x) \sim \begin{bmatrix} \varphi(b) + \chi(b) \\ \varphi(a) - \chi(a) \end{bmatrix} = \hat{T}_2 \begin{bmatrix} \varphi(b) - \chi(b) \\ \varphi(a) + \chi(a) \end{bmatrix}$$

$$\text{Dom}(\hat{h}_3 = -i\hbar c \hat{\beta} \partial_x) \sim \begin{bmatrix} \varphi(b) \\ \chi(a) \end{bmatrix} = \hat{T}_3 \begin{bmatrix} \chi(b) \\ \varphi(a) \end{bmatrix}$$

$$\text{Dom}(\hat{h}_4 = -i\hbar c i\hat{\beta}\hat{\alpha} \partial_x) \sim \begin{bmatrix} \varphi(b) + i\chi(b) \\ i\varphi(a) + \chi(a) \end{bmatrix} = \hat{T}_4 \begin{bmatrix} i\varphi(b) + \chi(b) \\ \varphi(a) + i\chi(a) \end{bmatrix}$$

- ▶  $\hat{T}_1^{-1} = \hat{T}_1^\dagger$ , etc
- ▶ Ejemplos :  $\varphi(a) = \varphi(b) = 0$ ,  $\chi(a) = \chi(b) = 0$ ,  $\psi(a) = \psi(b)$ ,  $\psi(a) = -\psi(b)$ . Nota :  $\psi(a) = \psi(b) = 0$  NO está presente

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