Appendix A

Exercise A.1: Verify that $T^{AB} \equiv -\frac{i}{2}\Gamma^{AB}$ are generators of $SO(1,D-1)$ in the spinor representation, i.e.

$$i[T^{AB},T^{CD}] = \eta^{AC}T^{BD} - \eta^{AD}T^{BC} - \eta^{BC}T^{AD} + \eta^{BD}T^{AC} \ .$$

Exercise A.2: Show that $\Gamma_{D+1}^2 = \mathbb{1}$, $\{\Gamma_{D+1},\Gamma^A\} = 0$, and $[\Gamma_{D+1},\Gamma^{AB}] = 0$.

Appendix C

Exercise C.1 : Use the transformation properties:

$$\mathcal{T} : \tau \to \tau + 1 \quad ; \quad \eta \to e^{\frac{i\pi}{12}}\eta \quad ; \quad \vartheta[^{\alpha}_{\beta}] \to e^{-i\pi(\alpha^2-\alpha)}\vartheta[^{\alpha}_{\alpha+\beta-\frac{1}{2}}]$$

$$\mathcal{S} : \tau \to -1/\tau \quad ; \quad \eta \to (-i\tau)^{\frac{1}{2}}\eta \quad ; \quad \vartheta[^{\alpha}_{\beta}] \to (-i\tau)^{\frac{1}{2}}e^{2i\pi\alpha\beta}\vartheta[^{\beta}_{-\alpha}]$$

to show that

$$\mathcal{Z}_F(\theta^k,\theta^\ell) = \frac{1}{4}\left|\sum_{\alpha,\beta=0,\frac{1}{2}} s_{\alpha\beta}(k,\ell)\,\frac{\vartheta[^{\alpha}_{\beta}]}{\eta}\prod_{j=1}^{3}\frac{\vartheta[^{\alpha+kv_j}_{\beta+\ell v_j}]}{\eta}\right|^2 \ .$$

has the correct modular transformations, i.e. $\mathcal{Z}_F(\theta^k,\theta^\ell)$ transforms into $\mathcal{Z}_F(\theta^k,\theta^{k+\ell})$ under $\mathcal{T}$ and into $\mathcal{Z}_F(\theta^\ell,\theta^{-k})$ under $\mathcal{S}$. The spin structure coefficients are:

$$s_{00}(k,\ell) = -s_{\frac{1}{2}0}(k,\ell) = 1 \quad ; \quad s_{0\frac{1}{2}}(k,\ell) = -s_{\frac{1}{2}\frac{1}{2}}(k,\ell) = -e^{-i\pi k(v_1+v_2+v_3)}$$